A COMPARISON OF DURBIN TWO-STEP FGLS AND FIRST-ORDER LAG MODELS IN FORECASTING

한국지방행정연구원

Ⅰ. INTRODUCTION

One of the central aims of research in econometrics or social sciences is to define basic principles whereby available information may be used to forecast future economic and social conditions or parameters. This paper presents an example of how quantitative methods can be applied in forecasting research.[0] To more effectively demonstrate the ability of forecast, it employs the real retail pork price data that could be obtained through USDA.[1]

In the following, the step by step development of two alternative price models is outlined. These alternatives result from the statistical weakness observed in simple OLS pre-test model which was initially investigated. The revised models prescribed here, a restricted first-order lag model and a Durbin two-stage feasible generalized least squares model, are derived from two different assumptions about the origin of the statistical problems in the simple OLS model. Even though they make these different assumptions about the source of error, the specifications of both the lag and FGLS models originate from an identical, more general structure, which was used as a pretest for each of these final models.

Ideally one of these models would be preferred in part due to better statistical properties and at least partially on the basis of some knowledge about the pork economy and relationships within meat markets. In this investigation, however, after the models were specified, they were compared and evaluated primarily on the basis of their forecast abilities. The same general strategy employed to arrive at and assess these pork price forecast models can be used in a wide spectrum of other research issues in numerous fields of study.

Ⅱ. MODEL DEVELOPMENT

Financial decisions or policies are often dependent upon the expected levels of commodity prices. For this reason, over the past several decades much work has been done to develop and refine methods and models for predicting and price trends in markets for agricultural products over time (Buse, 1989). A complete, systematic solution to this problem for the meat market would characterize and incorporate the interrelated nature of the entire industry by considering both supply and demand markets within the price forecast models (Harlow, 1962). Using

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1) This paper utilizes pork price data to better illustrate the basics of models developed here. The conclusion made here is expected to bring various applications in public sector research such as forecasting revenues and expenditures of local governments.

2) The material is contained in Livestock and Poultry Situation and Outlook, LPS-28, 1988 and USDA Agricultural Econ. Rep. 623.

the type of approach outlines by Harlow, product prices would be found by solving a set of equations where the prices and consumption quantities would be simultaneously determined and related to each other as well as to supply quantities.

While a simultaneous equations methodology has been a popular area of research and may provide a fruitful approach for modeling the meat economy in the United States, this study investigates more traditional, single equation models to forecast retail pork price. In this study consumption quantities of pork and other meats are treated as being exogenously determined. This approach may be justified in case retail meat prices are assumed to have no strong influence from supply side quantities. In reality, since meat supply quantities are almost entirely determined by decisions made at least two quarters in the past, this assumption is probably valid, at least in the short run. Past empirical results have shown this strategy[2] to result in relatively accurate forecasts for prices in the retail meat market (Buse, 1989)

Ⅲ. ORDINARY LEAST SQUARES MODEL AND EVALUATION

The first step in the development of this forecast was to identify factors deemed to have some potential influence on or relationship with the price of pork. Aside from consumption variables for pork and related meat products several other variables were initially considered. The independent variables used in the first regression included quarterly data describing the per capita consumption of pork, chicken, beef, and turkey. Potential seasonal variation in the meat economy was considered by including three quarterly dummy variables. In addition, the consumer price index was included as a proxy for the overall climate of the economy. The following variables were used in this model:

Dependent Variables:
RPPKt = retail price per pound of pork (deflated cents/lb)
Independent Variables:
CBFt = per capita consumption of beef (pounds/psn)
CCHt = per capita consumption of chicken (pounds/psn)
CTKt = per capita consumption of turkey (pounds/psn)
CPKt = per capita consumption of pork (pounds/psn)
Qi = seasonal dummy variable (i = 1, 2, 3 quarters)
CPIt = consumer price index (1982-1984 = 100)
Quarterly observations on all variables were used throughout the sample period

3) The strategy treats consumption quantities as exogenous.

which ranged from the first quarter of 1976 to the fourth quarter of 1992 (including a total of 68 points in the time series). Three quarters of 1993 data were withheld from the analysis in order to allow an ex-post forecast evaluation. As a pretest tool, an ordinary least squares regression was carried out using these variables. Results of the OLS model for the real price of pork are given in Table 1.

Table 1. Simple OLS Pre-Test Model Results

Predictor	Coef	Stdev	t-ratio	p	VIF
Constant	577.93	56.92	10.15	0.000	
CBF	-5.189	1.448	-3.58	0.001	10.3
CTK	2.887	3.821	0.76	0.453	32.6
CCH	6.667	2.247	2.97	0.004	23.2
CPK	-17.209	1.579	-10.90	0.000	2.8
CPI	-170.16	14.11	-12.06	0.000	15.7
Q1	-8.60	10.96	-0.78	0.436	27.6
Q2	-19.19	11.23	-1.71	0.093	29.0
Q3	-12.706	9.203	-1.38	0.173	19.5

s = 7.445 R-sq. = 0.935 R-sq(adj) = 0.927 D-W = 0.89

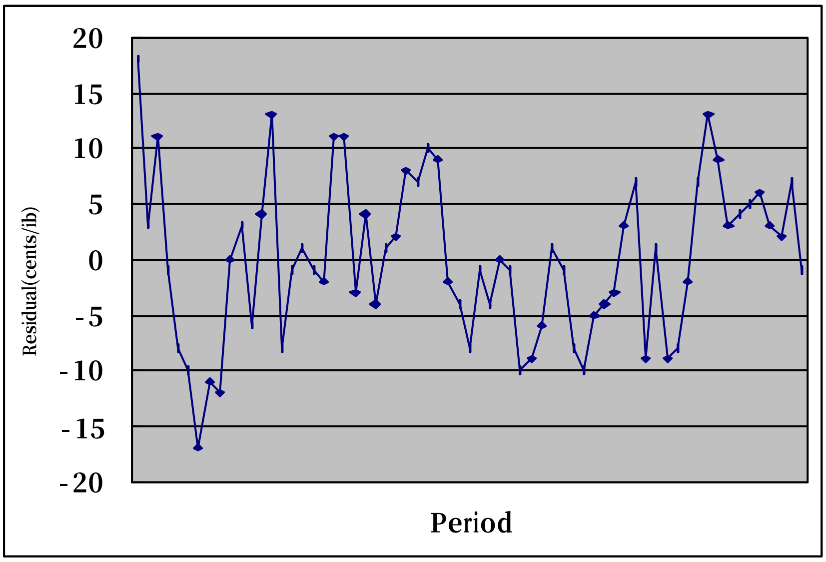
Aside from the economically illogically signs of the coefficients for turkey and chicken consumption[5)] several statistical problems with this model can be observed, reflecting on its quality. First, it is seen that the regressors for the three seasonal dummy variables are apparently not significant in this model. In addition there is a large degree of collinearity in the regressors. This is observed through the magnitude of the variance inflation factors (VIF) in the Table 1. A similar conclusion may be reached by observing the large magnitudes of many of the elements in the variance-covariance matrix for the regressors. Due to this collinearity in the regressors their variances illustrated in Table 1 for the OLS regression generate the biased variances. This may not be a large problem, however, for a forecast model. If multicollinearity was the only problem with this model and since in that case these coefficients would not be statistically biased, the pork price forecast would under those circumstances be similarly unbiased. The purpose of this model is to forecast pork price and not necessarily to obtain the true pork price model. Therefore, a problem of pure collinerity would not be a major concern for this application since there is no reason to believe structure of the multicollinerity seen here will not persist into the future. In that case an error

4) These goods should be substitutes for pork.

in one direction in one coefficient will tend to be balanced by an error in a second coefficient, leaving the forecast variable unbiased.

A more serious problem with this model is indicated by the small magnitude of the Durbin-Watson statistic. This points to either one of two potential problems, autocorrelation or model misspecification. Autocorrelation occurs with time-series data because something associated with time, such as growth, has not been taken into account and that thing influences several observations. Figure 1 illustrates the type of time-dependent pattern typical of residuals of a model with autocorrelated errors. Visual inspection of the residuals of the above OLS model plotted against time shows that positive residuals tend to be followed by other positive residuals, negative residuals by negative residuals, a sign of autocorrelated errors.

Figure 1. Simple OLS Regression: Residuals vs. Time



Similarly, with eight regressors in the model (k=9) and a Durbin-Watson statistic equal to 0.89, the hypothesis that there is no autocorrelation must be rejected at 1% level of significance.

The implications of this apparent autocorrelation are that although the estimate of the ordinary least squares regression coefficients are unbiased, there will be an downward bias in their variances. The OLS forecast model will therefore be inefficient and will lead to a non-optimal forecast model at least for pork price.

It is important to note that the above comments about the quality of the OLS model assume that it is correctly specified or at least that it contains all relevant regressors in the true model. If the model does not include one or more relevant regressors, the result may be a biased price forecast. In this event the low Durbin-Watson statistic, rather than indicating autocorrelation, could equally well be a sign of one or more relevant variables being omitted from the model specification (Kennedy, 1993).

In the following two sections separate models are developed and investigated which may be used to correct for simple first-order autocorrelation and misspecification in the above OLS formulation.

Ⅳ. FGLS MODEL FOR AUTOCORRELATED ERRORS

If first-order autocorrelated and not model misspecification is in fact the case, a solution can be based upon some type of feasible generalized least squares technique. For this application the Durbin's Two Stage Method was selected. The reason for this choice is that a DTSM-type regression is robust in that an identical model can be used to resolve autocorrelation in the error terms of a properly specified model as to test for misspecification due to lack of first-order lag terms in the original model. The theoretical background supporting this assertion is discussed in more detail below.

The assumed functional form of a true model with first order autocorrelated errors is:

(1)

Lagging by one period and multiplying the entire equation by results in:

(2)

Subtracting these two relations and substituting for the difference between the error and lagged error terms gives the model to be estimated in the first step of the Durbin Two-Stage Model:

(3)

Since the error term in this equation is assumed to be normally distributed, it is proper to estimate this functional form directly using ordinary least squares. The OLS-estimated coefficient for the lagged endogenous variable in this case is the DTSM feasible generalized least squares estimate for the first-order autocorrelation coefficient . Although this can be shown to be a biased estimate of the coefficient, it is however consistent for large samples (Kennedy, 1993).

The regression results for the first stage of the DTSM procedure are shown in Table 2. In this model the terms LCBF, LCTK, LCCH, LCPK, LCPI and LRPPK are regressors lagged one quarter relative to those defined in the original model. The coefficient for the lagged pork price gives an estimated autocorrelation coefficient of 0.5285.

Table 2. Unrestricted First-Order Lag Model OLS Regression

Predictor	Coef	Stdev	t-ratio	p	VIF
Constant	335.30	71.40	4.70	0.000	
CBF	-1.710	1.353	-1.26	0.212	21.6
LCBF	-1.257	1.233	-1.02	0.313	18.0
CTK	-1.266	3.716	-0.34	0.735	74.0
LCTK	0.465	3.582	0.13	0.897	62.7
CCH	2.018	2.291	0.88	0.382	57.9
LCCH	-0.551	2.370	-0.23	0.817	62.6
CPK	-13.879	1.711	-8.11	0.000	7.9
LCPK	2.538	2.596	0.98	0.333	19.0
CPI	260.1	154.1	1.69	0.097	4489.4
LCPI	-318.9	148.6	-2.15	0.036	4196.8
Q1	-20.03	15.05	-1.33	0.189	124.9
Q2	-23.274	8.274	-2.81	0.007	37.7
Q3	-14.904	6.992	-2.13	0.038	26.9
LRPPK	0.52853	0.08847	5.97	0.000	20.3

s = 4.810 R-sq. = 0.976 R-sq.(Adj) = 0.969 D-W = 1.65

To arrive at the DTSM pork price forecast model, the estimated autocorrelation coefficient was then used to transform the original data matrix. In this second stage of the DTSM procedure another OLS-type regression was run using as exogenous variables the original variables transformed by replacing by . Normally in the DTSM, to avoid losing an observation the first observation is transformed to . However, in this case since 1975 quarterly data was available, fourth quarter data was used to compute the lagged first terms in the transformed data matrix. It should be noted that in the transformed regression, since they are not constant factors which merely shift the y-intercept term, the form of the seasonal dummy variables remains as before.

The resulting regression output gives estimated values in the above equations. The result for the second stage regression obtained after transforming the data matrix as described above are shown in Table 3. The prefix "T" on these indicates they are coefficients for the transformed variables. An F-test evaluation of this regression results indicated that the transformed beef and turkey consumptions variables were not significant in the model at a 95% levels. If true, this result implies an over-specified model with an upwardly biased variance in the model's regressors.

Table 3. Stage 2, Durbin Two-Stage FGLS Model Regression Results

(estimated = 0.5285)

Predictor	Coef	Stdev	t-ratio	p	VIF
Constant	216.432	24.96	8.67	0.000	
TCBF	-1.588	1.294	-1.23	0.225	4.4
TCCH	6.624	2.164	3.06	0.003	10.4
TCTK	1.562	3.653	0.43	0.670	54.2
TCPK	-14.081	1.596	-8.82	0.000	3.1
TCPI	-139.952	17.75	-7.88	0.000	9.8
Q1	-16.827	14.27	-1.18	0.243	83.6
Q2	-21.925	9.176	-2.39	0.020	34.6
Q3	-11.319	7.253	-1.56	0.124	21.6

$s = 5.573$ R-sq. $= 0.847$ R-sq.(Adj) $= 0.826$ D-W $= 1.44$

Since the estimated autocorrelation coefficient was solved for with an improperly specified model, it is necessary to re-estimate the autocorrelation coefficient using the reduced form of the model shown in Table 2.[4] The new estimated autocorrelation coefficient in this case was 0.6122. Results of the second stage of the DTSM run using this value are shown in Table 4. Statistically, this model appears satisfactory; with low t-ratios and low variance inflation factors for each of the regressors. The results of this model were transformed to obtain the DTSM real pork price forecasts by adding the estimated autocorrelation coefficient times the lagged real pork price to the estimated dependent variable in this model. Forecasting results and an evaluation of the forecasting quality of this model are discussed in the final section of this paper.

Table 4. Durbin Two-Stage FGLS Restricted Model Regression Results

(estimated = 0.6122)

Predictor	Coef	Stdev	t-ratio	p	VIF
Constant	163.876	9.289	17.64	0.000	
TCCH	6.471	2.109	3.07	0.003	8.0
TCPK	-13.443	1.519	-8.85	0.000	3.0
TCPI	-121.662	16.75	-7.26	0.000	6.3
Q1	-23.293	3.395	-6.86	0.000	5.0
Q2	-25.370	3.858	-6.58	0.000	6.5
Q3	-14.295	3.058	-4.68	0.000	4.1

s = 5.405 R-sq. = 0.799 R-sq.(Adj) = 0.780 D-W = 1.45

V. FIRST-ORDER UNRESTRICTED LAG MODEL

The regression results shown in Table 2 may alternatively be interpreted another way, as an unrestricted first-order lag model. The form of this regression is:

In fact the results of the DTSM procedure, which assumed a model specification identical to the original OLS model above but with first-order autocorrelated errors,

4 The reduced form of the model removes CBF, LCBF, CTK and LCTK as regressors.

is just one restricted case of this equation. Other types of restrictions on a lag model of this general form may be used to test hypothesis about the mechanisms influencing the dynamic behavior of the pork price with time. Placing various restrictions on this model can test whether the dynamic behavior is generated by a partial adjustment relationship, a geometric form distributed lag, a dead-start type relationship, or one of several other mechanisms (Hendry, 1984). Although it is beyond the scope of this paper to investigate mechanisms within pork markets which may result in one or more of these restricted functional forms being preferred, these alternative specifications may be worthy of further study.

Some observations can be made about the results of this regression when it is viewed as a lag model. Although there continues to be collinearity, as discussed above this on the whole will not tend to bias the forecast prices and thus may not be a serious problem in this application. Several t-ratios in Table 2, however, lead to a hypothesis that there still exist some insignificant regressors in that model. If true, this would indicate a misspecified model. An F-test was carried out in order to test for the significance of the variables CCH, LCCH, CTK, LCTK, LCBF and LCPK. The null hypothesis in this case was that all of these coefficients equal zero.[5] Since the null hypothesis could not be rejected at 5% significance, these constraints were added to the model. The final pork price first-order lag model for that was used for forecasts is given in Table 5.

Table 5. Restricted First-Order Lag Model OLS Regression Results

Predictor	Coef	Stdev	t-ratio	p	VIF
Constant	357.60	42.76	8.36	0.000	
CBF	-2.955	1.028	-2.87	0.006	12.9
CPK	-12.204	1.211	-10.08	0.000	4.1
CPI	304.9	141.7	2.15	0.036	3927.1
LCPI	-358.1	138.0	-2.60	0.012	3741.1
Q1	-10.421	2.006	-5.20	0.000	2.3
Q2	-15.102	2.190	-6.89	0.000	2.7
Q3	-7.800	2.348	-3.32	0.002	3.1
LRPPK	0.48469	0.04532	10.69	0.000	5.5

s = 4.731   R-sq. = 0.974   R-sq.(Adj) = 0.970   D-W = 1.55

5 As opposed to at least one coefficient unequal to zero.

The statistical properties of this final model appear to be satisfactory with large t-ratios and low VIF's for all the regressors. The removal of several insignificant regressors from the model has greatly reduced the VIF's indicating far less collinearity in this model.[6] The forecast quality of this first-order lag model is discussed below.

Ⅵ. FORECAST EVALUATIONS

The two model formulations implied by the DTSM Feasible Generalized Least Squares and first-order lag models are, respectively:

where

Illustrations of the forecast results for these two models are shown in Figure 2 and 3. It should be noted that in order to fully evaluate the statistical properties of these models to determine whether there has been some misspecification error, a series of tests should be carried out (McGuirk, 1993). For this application we will be more interested in an evaluation based on the relative forecast quality of two models.

6 CPI and LCPI which are collinear with one another are an exception.

Figure 2. Durbin Two-Step FGLS Figure 3. First-Order Lag Model

Two procedures were carried out to compare the forecasting ability of these models. First, the accuracy of the models in predicting turning points in the data was evaluated using an ex-ante tracking procedure. This test looks at the actual and forecast directions of pork price changes between consecutive periods beginning with the first quarter of 1976 and running to the final quarter of 1992. Second, an ex-post forecast procedure was used to compare these models and gauge their accuracy with respect to a naive model which simply forecasts the previous year's pork price as the next year's price. The ex-ante and ex-post forecast evaluations of the two models are given below.

To measure the models' abilities to forecast turning points a relative turning points error evaluation was applied to the two forecast models. These calculations show that the lag model correctly predicted 53 out of 66 total changes (80%), outperforming the DTSM model which predicted 48 out of 66 (73%). Of the 26 turning points in the sample, the lagged model performed slightly better than the DTSM model, with 23 (88%) compared to 22 (85%) predicted correctly.

Theil's inequality coefficient ( ) was used to compare these models' ex-post forecasting abilities. The coefficient equals the ratio of the sum of the squared deviations of the actual from forecast price changes to the sum of the squared actual price changes. A model which naively forecasts the price from the previous period would therefore have a coefficient equal to one. A coefficient less than one indicates a model which performs better than this naive model. Results using values from the first three quarters of 1993 indicate values for the DTSM model performed somewhat worse than a naive forecast of changed real pork price and the lag model performed slightly better.

Ⅶ. CONCLUSION

Both of the pork price forecast models developed in this paper appear to give relatively accurate predictions. The indicated statistical accuracy of these results, however, are dependent upon several inherent assumptions including proper model specification, homoscedasticity and uncorrelated errors.[7] These possibilities have not been fully investigated in this analysis.

It should not be too surprising that the lag model somewhat outperformed the DTSM model, especially in the ex-ante forecast evaluation. This is because the DTSM may alternatively be viewed as a restricted case of the initial unrestricted first-order lag model from which the final lag model was derived. The assumption for the DTSM was that a simple first-order autocorrelation mechanism determines the pork price. There are also, however, several other conceivable mechanisms which may be more realistic in explaining how pork price is determined. It is beyond the scope of this paper to investigate mechanisms in the economy which determine retail pork price in the short run, but it is noted that many realistic mechanisms including partial adjustments, simple and geometric form distributed lags, first differences and others may be described mathematically through different types of restrictions on the original lag model (Hendry, 1984). Some of these may be worthy of investigation in future work aimed at understanding the dynamics at work in the pork market.

The lag model developed was similarly derived from an unrestricted first-order form. On the basis of the evaluations in this investigation this model appears preferable to the DTSM as a forecasting tool. Some warning about this method of model development, however, is warranted. As opposed to basing improvements and restrictions on hypotheses about realistic mechanisms which determine the model form, the form in this case was determined simply by the statistical properties of the unrestricted model. It will therefore naturally place fewer restrictions on the final form than the DTSM model. For example, in the lag model the significance of current and lagged independent variables were considered individually. Similar determinations in the case of a model based on an autocorrelated error structure in effect must made in pairs.[8] In addition, the DTSM framework in effect restricts the ratio of relative sizes of the current and lagged endogenous variables for a given regressor to be a constant. No such restriction is made in the lag model. Therefore, this method places fewer restrictions on the model form and will generally result in a final model better correlated with the sample data.

The results derived on the basis of this type procedure may, however, not be

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7 The uncorrelated errors are in the case of the lag model.

9) It will discard the transformed variable which is related to both current and lagged variables.

theoretically satisfying since it is not based on any theory about real economic mechanisms. In cases where some extra knowledge about the economic system indicates there is a likely "true" mechanism by which prices are determined, restrictions to that effect may be desirable. This holds true even if the statistical properties of a limited-size sample seem to indicate an alternate but ad-hoc model is preferred. In this case the lagged model is preferred on account of its somewhat better statistical properties and forecasting ability since we have no additional information favoring an autoregressive form over any other.

The discussion made in this paper can be applied to other issues such as revenue forecasting, expenditure forecasting for local governments. The revenue consisting of local taxes, transfers, and bond issuance is based on various factors including an economic environment. For example, tax money can be determined by tax rate and the size of tax base. The size of tax base is related to the economic condition and demographics in the region. The basic principle of forecasting the tax revenue could be the same as the logic that the future pork price is estimated. The application to those fields should be worthy of extra efforts in future work.

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Taeyoung Kim is a senior research fellow of Korea Research Institute for Local Administration in Korea. His researchinterests include financial analysis of local governments, quantitative methods, public and urban economics, city management and public policies.